December 2000 PAR-LPTHE 00/48

GAUGE BOSONS IN AN $\mathbf{SU(2)_L \times SU(2)_R} \times G_{lept}$ ELECTROWEAK MODEL

B. Machet [1] [2]

Laboratoire de Physique Théorique et Hautes Énergies [3]
Universités Pierre et Marie Curie (Paris 6) et Denis Diderot (Paris 7)
Unité associée au CNRS UMR 7589

Abstract: By considering its generalization to composite $J=0$ mesons proposed in [1], I show how and why a chiral extension of the Glashow-Salam-Weinberg standard model of electroweak interactions calls, there, for right-handed charged $W^\pm_{\mu R}$'s coupled with $g_R = e/\cos\theta_W$, and the masses of which are related to the ones of the left-handed $W^\pm_{\mu L}$'s through the relation $M^2_{W_L} + M^2_{W_R} = M^2_Z$. The mesonic sector, having vanishing baryonic and leptonic number, is neutral with respect to the corresponding $U(1)_{\mathbb{I}}$ symmetries, making the natural chiral gauge group to be $SU(2)_L \times SU(2)_R$, blind to the presence of extra Z'_μ's. The $W^\pm_{\mu R}$ gauge bosons cannot have been detected in hadronic colliders and can be very elusive in electroweak processes involving, in particular, pseudoscalar mesons. Present data select one among two possible extensions for which, in the right sector: – a specific breaking of universality occurs between families of quarks, which belong to inequivalent representations of $SU(2)_R$; – the mixing angle is a free parameter, constrained to be smaller than the Cabibbo angle by the box diagrams controlling the $K_L - K_S$ mass difference; this also minimizes contributions to $\mu \to e\gamma$. The relation $g^2_L/M^2_{W^\pm_L} = g^2_R/M^2_{W^\pm_R}$ implements left-right symmetry for low energy charged effective weak interactions. For the sake of simplicity, this study is performed for two generations only.

PACS: 11.30.Rd 12.60.Cn 12.60.Rc 14.70.Fm

CENTRE NATIONAL
DE LA RECHERCHE
SCIENTIFIQUE

[1] Member of 'Centre National de la Recherche Scientifique'

[2] E-mail: machet@lpthe.jussieu.fr

[3] LPTHE tour 16 / 1^{er} étage, Université P. et M. Curie, BP 126, 4 place Jussieu, F-75252 Paris Cedex 05 (France)

1 Introduction.

The Glashow-Salam-Weinberg standard model of electroweak interactions accounts for parity violation by enforcing it from the start at the level of the gauge group of symmetry; one would rather like the asymmetry observed in nature to arise from dynamical considerations. This was one of the goals of left-right symmetric models, in particular the ones based on the gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ [2].

We too shall be concerned here with finding a chiral extension to the standard electroweak model; more precisely, we shall extend to a chiral form the already achieved extension of the standard model to $J = 0$ states transforming like fermion-antifermion composite fields [1] .

The main step is to identify the weak hypercharge generator $\mathbb{Y}$ with the right-handed $\mathbb{T}^3_R$ generator; this is possible for mesons because the extra purely vectorial $U(1)_{\mathbb{I}}$ part of $U(1)_{\mathbb{Y}}$ does not act on composite fermion-antifermion fields; essential at the fermionic (leptonic) level, it is of no relevance as far as the spectrum of the gauge fields, entirely determined at the hadronic level by the vacuum expectation values of composite mesonic fields, is concerned. Hence, the left-right symmetric extension that we are led to consider here involves only the group $SU(2)_L \times SU(2)_R$, with left and right coupling constants being respectively $e/\sin\theta_W$ and $e/\cos\theta_W$, as suggested by the Gell-Mann-Nishijima relation written in chiral form; the left-right symmetry [3], absent at the level of the Lagrangian, is however implemented at low energy for effective charged weak interactions.

Both the left- and right- $SU(2)$'s have the same generator $\mathbb{T}^3$: it is a constraint from which to build the sought for $SU(2)_R$ group; the diagonal $U(1)$ with generator $\mathbb{T}^3$ is the only one which can be left unbroken in the spontaneous breaking of the electroweak symmetry; it corresponds to the electric charge in the mesonic sector.

The electroweak symmetry breaking is triggered by the non-vanishing vacuum expectation value of a scalar(s) meson(s) (Higgs boson), which behaves itself like a quark-antiquark composite; all $J = 0$ composite representations depend on the mixing angles and are, in the case of $SU(2)_L$, isomorphic to the scalar quadruplet of the standard model; their $SU(2)_R$ counterparts are easily determined but, in general, none is a rep. of both $SU(2)_L$ and $SU(2)_R$ such that the $SU(2)_L \times SU(2)_R$ symmetry breaking scalar potential has to make use of appropriate combinations of them. No Higgs multiplet with non-vanishing lepton $(B - L)$ number [4] is introduced.

We are concerned in this work with the spectrum of the gauge fields, constrained by the observed properties of the Z_μ and $W^\pm_{\mu L}$. The question of an extra Z'_μ does not arise for $SU(2)_L \times SU(2)_R$. The charged sector is enlarged by two $W^\pm_{\mu R}$ with masses $M_{W_R} = M_Z \sin\theta_W = M_{W_L}\tan\theta_W \approx 43\, GeV$.

There happens to be two possible right-handed $SU(2)_R$ groups; this is linked to the fact that the N-vector of fermions, which lies in the fundamental representation of the diagonal subgroup of the chiral $U(N)_L \times U(N)_R$, is reduceable with respect to the electroweak $SU(2)$'s and split into $N/2$ doublets, one for each family, which can belong to inequivalent representations.

In each of the two possible extensions there occurs, in the right sector, the equivalent φ of the Cabibbo angle θ_c of the left-handed sector; for the first type of $SU(2)_R$ it is constrained to be $\varphi = \theta_c + \pi/2$, while it is left free in the second type; in this last case, the couplings of the new gauge fields to mesons become φ-dependent; we shall see that it is the one favoured by data for pseudoscalar mesons. In general, none of the relations between left and right mixing angles commonly considered in the literature [2] appears to be realized in this approach: the right angles are free parameters to be determined by experiment like their left counterparts.

Light $W^\pm_{\mu R}$ like mentioned above have been unsought for, and we show in particular that they cannot have been detected in hadronic colliders, which started there investigations at a higher threshold.

They can eventually yield faint signals in electroweak processes, and we investigate the special decay $D^+ \to \pi^0 K^+ \pi^0$, seemingly undetected; explaining this absence by a cancellation between $W^\pm_{\mu L}$ and

[1] It includes from the start the chiral and electroweak properties of quarks (which are no longer fields of the Lagrangian).

$W^{\pm}_{\mu R}$ discards the first possible chiral extension and favors the one where, in the associated quark picture, the two families belong to inequivalent representations of $SU(2)_R$. Then, we show that the influence of $W^{\pm}_{\mu R}$ on the computation of the $K_L - K_S$ mass difference at the quark level can be made negligeable when the corresponding mixing angle is very small, without advocating for very massive $W^{\pm}_{\mu R}$ [5] .

Small enough right mixing angles also provides any desired suppression of unwanted contributions to $\mu \rightarrow e\gamma$.

This study does not pretend to be exhaustive nor to provide a universal chiral extension of the standard electroweak model; in particular, as stressed again in the conclusion, it can hardly be conceived that dealing with the leptonic sector does not require additional gauge field(s) and a more complex group structure. Nevertheless, a chiral extension of the standard model should accommodate the results presented here in the sector of composite $J = 0$ mesons.

We deal with $N/2 = 2$ generations of quarks.

2 $\mathbf{SU(2)_L \times U(1)}$ as a truncated $\mathbf{SU(2)_L \times SU(2)_R}$

While an abelian $U(1)$ group can be chirally non-diagonal, a non-abelian group can only be right-handed, left-handed or diagonal; our goal being to "extend" the hypercharge $U(1)_{\mathbb{Y}}$ group of the standard model to a non-abelian structure, a special attention to handedness is due.

Any $SU(2)$ (and $U(1)$) group can be considered as a subgroup of chiral $U(N)_L \times U(N)_R$ for N even, and its generators taken as $N \times N$ matrices.

Accordingly, we rewrite the Gell-Mann-Nishijima relation in its chiral form [1]

$$(\mathbb{Y}_L, \mathbb{Y}_R) = (\mathbb{Q}_L, \mathbb{Q}_R) - \mathbb{T}^3_L, \tag{1}$$

where, in the hadronic sector

$$\mathbb{Q} = \left(\begin{array}{c|c} \frac{2}{3}\mathbb{I}_2 & \\ \hline & -\frac{1}{3}\mathbb{I}_2 \end{array}\right), \quad \mathbb{T}^3 = \frac{1}{2}\left(\begin{array}{c|c} \mathbb{I}_2 & \\ \hline & -\mathbb{I}_2 \end{array}\right), \tag{2}$$

$\mathbb{I}_2$ being the 2×2 identity matrix.

(1) also rewrites

$$\mathbb{Y}_{hadr} = \frac{1}{6}\mathbb{I}_4 + \mathbb{T}^3_R \tag{3}$$

where $\mathbb{I}_4$ is the 4×4 identity matrix [2] .

For any 4×4 matrix $\mathbb{M}$, a composite state $\overline{\Psi}(\gamma_5)\mathbb{M}\Psi$ is left invariant by the action of $\mathbb{I}_4$ since it is modified [1] by the commutator $\overline{\Psi}(\gamma_5)[\mathbb{I}_4, \mathbb{M}]\Psi$ when Ψ and $\overline{\Psi}$ are acted upon. Hence, when dealing with composite $J = 0$ mesons (considered to be both the fields of the Lagrangian and the asymptotic states of the theory) the weak hypercharge coincides with the right-handed generator $\mathbb{Y}_{hadr} \equiv \mathbb{T}^3_R$. It is the starting point for building the looked for right handed $SU(2)_R$ [3] .

[2] In general, $\mathbb{Y} = \alpha\mathbb{I} + \mathbb{T}^3_R$, with $\alpha = 1/6$ for hadrons and $\alpha = -1/2$ for leptons; in the framework of a chiral theory where right-handed leptons are doublets of $SU(2)_R$, α is interpreted as $(B - L)$ [6].

[3] (3) also writes $\mathbb{Q} = \frac{1}{6}\mathbb{I}_4 + \mathbb{T}^3$, which shows that, in the mesonic sector, for the reasons just explained, the electric charge generator $\mathbb{Q}$ coincides with $\mathbb{T}^3$.

2.1 Two types of $\mathrm{SU}(2)$

According to the remarks above, we look for the possible $SU(2)_R$ groups, with generators taken as $N \times N \equiv 4 \times 4$ matrices, among which $\mathbb{T}^3$ is given in (2).
Let $\mathbb{D}_1$ and $\mathbb{D}_2$ be the two 2×2 matrices

$$\mathbb{D}_1 = \mathbb{I}_2 = \begin{pmatrix} 1 & \\ & 1 \end{pmatrix}, \quad \mathbb{D}_2 = \begin{pmatrix} 1 & \\ & -1 \end{pmatrix} \tag{4}$$

and $\mathcal{R}(\varphi)$ the rotation matrix

$$\mathcal{R}(\varphi) = \begin{pmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{pmatrix}, \tag{5}$$

to which we associate the 4×4 matrix

$$\mathbb{R}(\varphi) = \left(\begin{array}{c|c} \mathbb{I}_2 & \\ \hline & \mathcal{R}(\varphi) \end{array}\right). \tag{6}$$

The first $SU(2)$, $\mathcal{G}_1$ has generators

$$\begin{aligned}
\mathbb{T}_1^3 &= \frac{1}{2}\mathbb{R}^\dagger(\varphi) \left(\begin{array}{c|c} \mathbb{I}_2 & \\ \hline & -\mathbb{I}_2 \end{array}\right) \mathbb{R}(\varphi) = \frac{1}{2} \left(\begin{array}{c|c} \mathbb{I}_2 & \\ \hline & -\mathbb{I}_2 \end{array}\right) \equiv \mathbb{T}^3, \\
\mathbb{T}_1^+(\varphi) &= \mathbb{R}^\dagger(\varphi) \left(\begin{array}{c|c} & \mathbb{D}_1 \\ \hline & \end{array}\right) \mathbb{R}(\varphi) = \left(\begin{array}{c|c} & \mathcal{R}(\varphi) \\ \hline & \end{array}\right), \\
\mathbb{T}_1^-(\varphi) &= \mathbb{R}^\dagger(\varphi) \left(\begin{array}{c|c} & \\ \hline \mathbb{D}_1 & \end{array}\right) \mathbb{R}(\varphi) = \left(\begin{array}{c|c} & \\ \hline \mathcal{R}^\dagger(\varphi) & \end{array}\right).
\end{aligned} \tag{7}$$

and the second, $\mathcal{G}_2$, has generators

$$\begin{aligned}
\mathbb{T}_2^3 &\equiv \mathbb{T}_1^3 \equiv \mathbb{T}^3, \\
\mathbb{T}_2^+(\varphi) &= \mathbb{R}^\dagger(\varphi) \left(\begin{array}{c|c} & \mathbb{D}_2 \\ \hline & \end{array}\right) \mathbb{R}(\varphi) = \left(\begin{array}{c|cc} & \cos\varphi & \sin\varphi \\ & \sin\varphi & -\cos\varphi \\ \hline & & \end{array}\right), \\
\mathbb{T}_2^-(\varphi) &= \mathbb{R}^\dagger(\varphi) \left(\begin{array}{c|c} & \\ \hline \mathbb{D}_2 & \end{array}\right) \mathbb{R}(\varphi) = \left(\begin{array}{cc|c} & & \\ \hline \cos\varphi & \sin\varphi & \\ \sin\varphi & -\cos\varphi & \end{array}\right).
\end{aligned} \tag{8}$$

Transforming φ into $\varphi + \pi/2$ is equivalent to changing, in $\mathcal{G}_1$, $\mathbb{D}_1$ into

$$\mathbb{D}_4 = \begin{pmatrix} & 1 \\ -1 & \end{pmatrix} \tag{9}$$

and transforming φ into $\varphi+\pi/2$ in $\mathcal{G}_2$ equivalent to going from $\mathbb{D}_2$ to

$$\mathbb{D}_3 = \begin{pmatrix} & 1 \\ 1 & \end{pmatrix}. \tag{10}$$

The Glashow-Salam-Weinberg model uses $\mathcal{G}_1$ as the $SU(2)_L$ group acting on the 4-vector of quarks

$$\Psi = \begin{pmatrix} u \\ c \\ d \\ s \end{pmatrix}; \tag{11}$$

Ψ, reduceable with respect to $SU(2)$, lies in the fundamental representation of the diagonal $U(4)$ subgroup of the chiral group $U(4)_L \times U(4)_R$; $\mathcal{R}(\varphi=\theta_c)$ is then the Cabibbo matrix $\mathbb{C}$.

The eight $J=0$ composite representations of $SU(2)_L$ transforming like quark-antiquark operators are built according to [1];

$$\begin{aligned}
&\Phi_L(\mathbb{D},\theta_c) = [\mathbb{M}^0_L, \mathbb{M}^3_L, \mathbb{M}^+_L, \mathbb{M}^-_L](\mathbb{D},\theta_c) \\
= \;& \mathbb{R}^\dagger(\theta_c)\left[\frac{1}{\sqrt{2}}\left(\begin{array}{c|c} \mathbb{D} & 0 \\ \hline 0 & \mathbb{D}\end{array}\right), \frac{i}{\sqrt{2}}\left(\begin{array}{c|c} \mathbb{D} & 0 \\ \hline 0 & -\mathbb{D}\end{array}\right), i\left(\begin{array}{c|c} 0 & \mathbb{D} \\ \hline 0 & 0\end{array}\right), i\left(\begin{array}{c|c} 0 & 0 \\ \hline \mathbb{D} & 0\end{array}\right)\right]\mathbb{R}(\theta_c) \\
&= \left[\frac{1}{\sqrt{2}}\left(\begin{array}{c|c} \mathbb{D} & 0 \\ \hline 0 & \mathbb{C}^\dagger\,\mathbb{D}\,\mathbb{C}\end{array}\right), \frac{i}{\sqrt{2}}\left(\begin{array}{c|c} \mathbb{D} & 0 \\ \hline 0 & -\mathbb{C}^\dagger\,\mathbb{D}\,\mathbb{C}\end{array}\right), i\left(\begin{array}{c|c} 0 & \mathbb{D}\,\mathbb{C} \\ \hline 0 & 0\end{array}\right), i\left(\begin{array}{c|c} 0 & 0 \\ \hline \mathbb{C}^\dagger\,\mathbb{D} & 0\end{array}\right)\right],
\end{aligned} \tag{12}$$

with $\mathbb{D} \in \{\mathbb{D}_1 \cdots \mathbb{D}_4\}$, and split into the two types $(\mathbb{M}^0, \vec{\mathbb{M}}) = (\mathbb{S}^0, \vec{\mathbb{P}})$ and $(\mathbb{M}^0, \vec{\mathbb{M}}) = (\mathbb{P}^0, \vec{\mathbb{S}})$ where $\mathbb{S}$ denotes a scalar and $\mathbb{P}$ a pseudoscalar; their laws of transformations are given in [1]. They depend on the mixing angles and can be decomposed into two doublets (2) and $(\overline{2})$ of $SU(2)_L$, or one singlet plus one triplet of the diagonal custodial $SU(2)$. Each real quadruplet (12) is isomorphic to the complex scalar doublet of the standard model.

The meson field, of dimension [*mass*] attached to a given matrix is obtained [1] by sandwiching it between $\overline{\Psi}$ and Ψ, after eventually adding a γ_5 matrix for pseudoscalar mesons, and introducing an appropriate normalization factor [1][7]; there is in particular a one-to-one correspondence between the quark content of a meson and its matricial expression.

We shall consider for $SU(2)_R$ the two possibilities:

- $SU(2)_R = \mathcal{G}_1(\varphi)$, that we call the "replica" case because for $\varphi=\theta_c$, $SU(2)_R$ is the exact replica of the standard $SU(2)_L$; in particular, when the mixing angle is turned off, the two families of quarks doublets

$$\begin{pmatrix} u \\ d \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} c \\ s \end{pmatrix} \tag{13}$$

making up the reduceable quadruplet Ψ are acted upon in a similar way by $SU(2)$;

- $SU(2)_R = \mathcal{G}_2(\varphi)$, where the two families belong to inequivalent representations of $SU(2)_R$, thus exhibiting in this sector a specific breaking of universality; we call it the "inverted" case to remind that a "$-$" sign occurs relatively to the first family when $\mathbb{T}^\pm_{2R}$ act on the second family.

We shall study them successively; from the fact that $\{\mathbb{D}_1, \mathbb{D}_2, \mathbb{D}_3, \mathbb{D}_4\}$ form a complete set for real 2×2 matrices, one has exhausted, for the case of two generations, the possible extensions of the standard model for composite $J = 0$ mesons.

3 First case: $\mathbf{SU(2)_R}$ is the replica of $\mathbf{SU(2)_L}$

The $SU(2)_R$ generators are given by (7).

The eight $J = 0$ composite representations of $SU(2)_R$ (which can be decomposed into two doublets (2) and $(\overline{2})$) are built like for $SU(2)_L$; only the value of the mixing angle is different $\varphi \neq \theta_c$:

$$
\begin{aligned}
&\Phi_R(\mathbb{D}, \varphi) = [\mathbb{M}^0, \mathbb{M}^3, \mathbb{M}^+, \mathbb{M}^-]_R(\mathbb{D}, \varphi) \\
= \;& \mathbb{R}^\dagger(\varphi) \left[\frac{1}{\sqrt{2}} \left(\begin{array}{c|c} \mathbb{D} & 0 \\ \hline 0 & \mathbb{D} \end{array}\right), \frac{i}{\sqrt{2}} \left(\begin{array}{c|c} \mathbb{D} & 0 \\ \hline 0 & -\mathbb{D} \end{array}\right), i \left(\begin{array}{c|c} 0 & \mathbb{D} \\ \hline 0 & 0 \end{array}\right), i \left(\begin{array}{c|c} 0 & 0 \\ \hline \mathbb{D} & 0 \end{array}\right) \right] \mathbb{R}(\varphi) \\
&= \left[\frac{1}{\sqrt{2}} \left(\begin{array}{c|c} \mathbb{D} & 0 \\ \hline 0 & \mathcal{R}^\dagger(\varphi)\,\mathbb{D}\,\mathcal{R}(\varphi) \end{array}\right), \frac{i}{\sqrt{2}} \left(\begin{array}{c|c} \mathbb{D} & 0 \\ \hline 0 & -\mathcal{R}^\dagger(\varphi)\,\mathbb{D}\,\mathcal{R}(\varphi) \end{array}\right), \right. \\
&\qquad\qquad \left. i \left(\begin{array}{c|c} 0 & \mathbb{D}\,\mathcal{R}(\varphi) \\ \hline 0 & 0 \end{array}\right), i \left(\begin{array}{c|c} 0 & 0 \\ \hline \mathcal{R}^\dagger(\varphi)\,\mathbb{D} & 0 \end{array}\right) \right].
\end{aligned} \tag{14}
$$

Their laws of transformations are given in [1]. We use the same notation $\mathbb{S}$ for the scalar mesons, $\mathbb{P}$ for the pseudoscalars, and make explicit the matrix $\mathbb{D} \in \{\mathbb{D}_1, \mathbb{D}_2, \mathbb{D}_3, \mathbb{D}_4\}$ attached to the corresponding $(\mathbb{S}^0_R, \vec{\mathbb{P}}_R)$ or $(\mathbb{P}^0_R, \vec{\mathbb{S}}_R)$ quadruplets, together with the mixing angle φ. For example $\mathbb{P}^+_R(\mathbb{D}_3, \varphi)$ is the charged pseudoscalar $J = 0$ meson belonging to the $(\mathbb{S}^0_R, \vec{\mathbb{P}}_R)$ quadruplet of $SU(2)_R$ labelled by $\mathbb{D}_3$ and which depends on the mixing angle φ:

$$
\mathbb{P}^+_R(\mathbb{D}_3, \varphi) = i \left(\begin{array}{c|cc} & -\sin\varphi & \cos\varphi \\ & \cos\varphi & \sin\varphi \\ \hline & & \end{array}\right)_{pseudoscalar} ; \tag{15}
$$

the corresponding expression for $SU(2)_L$ is

$$
\mathbb{P}^+_L(\mathbb{D}_3, \theta_c) = i \left(\begin{array}{c|cc} & -\sin\theta_c & \cos\theta_c \\ & \cos\theta_c & \sin\theta_c \\ \hline & & \end{array}\right)_{pseudoscalar} . \tag{16}
$$

In general, unlike the Higgs multiplets introduced in other approaches [4], the reps. of $SU(2)_R$ are not reps. of $SU(2)_L$ (and vice-versa), the only exception occurring when $\varphi = \theta_c$.

3.1 The gauge Lagrangian

The kinetic terms for the mesons are built like in [1] by using the property of [4]

[4]Since throughout the paper we use a matricial *notation* for the fields, the symbol $\otimes$ has been used to mean the product of fields as space-time functions to avoid a possible misinterpretation with product of matrices.

$$
\begin{aligned}
\mathcal{J} &= (\mathbb{S}^0,\vec{\mathbb{P}})(\mathbb{D}_1)\otimes(\mathbb{S}^0,\vec{\mathbb{P}})(\mathbb{D}_1)+(\mathbb{S}^0,\vec{\mathbb{P}})(\mathbb{D}_2)\otimes(\mathbb{S}^0,\vec{\mathbb{P}})(\mathbb{D}_2)\\
&+ (\mathbb{S}^0,\vec{\mathbb{P}})(\mathbb{D}_3)\otimes(\mathbb{S}^0,\vec{\mathbb{P}})(\mathbb{D}_3)-(\mathbb{S}^0,\vec{\mathbb{P}})(\mathbb{D}_4)\otimes(\mathbb{S}^0,\vec{\mathbb{P}})(\mathbb{D}_4)\\
&- (\mathbb{P}^0,\vec{\mathbb{S}})(\mathbb{D}_1)\otimes(\mathbb{P}^0,\vec{\mathbb{S}})(\mathbb{D}_1)-(\mathbb{P}^0,\vec{\mathbb{S}})(\mathbb{D}_2)\otimes(\mathbb{P}^0,\vec{\mathbb{S}})(\mathbb{D}_2)\\
&- (\mathbb{P}^0,\vec{\mathbb{S}})(\mathbb{D}_3)\otimes(\mathbb{P}^0,\vec{\mathbb{S}})(\mathbb{D}_3)+(\mathbb{P}^0,\vec{\mathbb{S}})(\mathbb{D}_4)\otimes(\mathbb{P}^0,\vec{\mathbb{S}})(\mathbb{D}_4)
\end{aligned}
\tag{17}
$$

to be invariant both

- by $SU(2)_L$ when the quadruplets $(\mathbb{S}^0,\vec{\mathbb{P}})$, $(\mathbb{P}^0,\vec{\mathbb{S}})$ are chosen to be the representations of $SU(2)_L$, thus expressed in terms of θ_c;
- by $SU(2)_R$ when the quadruplets $(\mathbb{S}^0,\vec{\mathbb{P}})$, $(\mathbb{P}^0,\vec{\mathbb{S}})$ are chosen to be the representations of $SU(2)_R$, thus expressed in terms of φ.

The two expressions for $\mathcal{J}$ can indeed be seen to be identical in the basis of quark ”flavour eigenstates” $\bar q_i q_j$ and $\bar q_i \gamma_5 q_j$ in which all dependence on the mixing angle vanishes [1].

Consequently, the kinetic terms for the mesons are obtained from $\mathcal{J}$ by replacing each mesonic field by the corresponding covariant derivative with respect to the $SU(2)_L \times SU(2)_R$ gauge group, and they can equally be expressed using $SU(2)_L$ or $SU(2)_R$ quadruplets.

One introduces accordingly two sets of gauge bosons $\vec{W}_{\mu L}$ and $\vec{W}_{\mu R}$, and call the two corresponding coupling constants g_L and g_R.

3.2 The neutral gauge bosons

Neutral gauge bosons eventually get masses through kinetic terms $\frac{1}{2}\, D_\mu \mathbb{P}^3 D^\mu \mathbb{P}^3$, where D_μ is hereafter the covariant derivative with respect to $SU(2)_L \times SU(2)_R$; indeed, the axial part of the neutral gauge group generators, when acting on any neutral pseudoscalar $\mathbb{P}^3$, transforms it into a neutral scalar [5] ; if the latter gets a non-vanishing vacuum expectation value (see subsection 3.5 below), a mass term arises for the corresponding neutral gauge field.

Though all four $(\mathbb{S}^0,\vec{\mathbb{P}})(\mathbb{D}_{1\cdots 4},\theta_c)$ quadruplets are isomorphic to the complex scalar doublet of the standard model, it is specially convenient, to ease the computations, to choose the Higgs boson H to be $\mathbb{S}^0(\mathbb{D}_1)$ or $\mathbb{S}^0(\mathbb{D}_4)$, which do not depend on the mixing angles; this is also the case for the corresponding $\mathbb{P}^3$'s.

If one furthermore chooses H to be CP-even, this restricts the Higgs boson to $H = \mathbb{S}^0(\mathbb{D}_1)$.

From

$$
\begin{aligned}
D_\mu \mathbb{P}_3(\mathbb{D}_1) &\ni \left(\partial_\mu - i g_L W^3_{\mu L}\mathbb{T}^3_L . - i g_R W^3_{\mu R}\mathbb{T}^3_R .\right)\mathbb{P}_3(\mathbb{D}_1) + \ldots\\
&= \partial_\mu \mathbb{P}_3(\mathbb{D}_1) - \frac{1}{2}(g_L W^3_{\mu L} - g_R W^3_{\mu R}) H + \ldots
\end{aligned}
\tag{20}
$$

and

$$
< H > = < \mathbb{S}^0(\mathbb{D}_1) > = \frac{v}{\sqrt{2}}
\tag{21}
$$

[5] The action of a right generator on a pseudoscalar described by the matrix $\mathbb{M}$ ($[,]$ stands for the commutator and $\{,\}$ for the anticommutator) is

$$
\mathbb{T}^i_R.\mathbb{M}_{pseudoscalar} = \frac{1}{2}\left([\mathbb{M},\mathbb{T}_i]_{pseudoscalar} + \{\mathbb{M},\mathbb{T}^i\}_{scalar}\right)
\tag{18}
$$

and the action of a left generator

$$
\mathbb{T}^i_L.\mathbb{M}_{pseudoscalar} = \frac{1}{2}\left([\mathbb{M},\mathbb{T}_i]_{pseudoscalar} - \{\mathbb{M},\mathbb{T}^i\}_{scalar}\right)
\tag{19}
$$

one straightforwardly gets the spectrum of the neutral gauge fields:
- a massless photon

$$A_\mu = \sin\theta_W W^3_{\mu L} + \cos\theta_W W^3_{\mu R} \tag{22}$$

- a massive Z_μ

$$Z_\mu = \cos\theta_W W^3_{\mu L} - \sin\theta_W W^3_{\mu R} \tag{23}$$

with mass

$$M_Z^2 = \frac{g^2 v^2}{16\cos^2\theta_W}, \tag{24}$$

and the usual relations

$$g_L = \frac{e}{\sin\theta_W},\ g_R = \frac{e}{\cos\theta_W},\ e = \frac{g_L g_R}{\sqrt{g_L^2 + g_R^2}} \tag{25}$$

where e is the unit electric charge.
In addition to the standard $Z_\mu - W^+_{\nu L} - W^-_{\rho L}$ coupling proportional to $e\cos\theta_W/\sin\theta_W$, there exists now a coupling $Z_\mu - W^+_{\nu R} - W^-_{\rho R}$ proportional to $e\sin\theta_W/\cos\theta_W$, that is smaller than the previous one by a factor $\tan^2\theta_W \approx .28$ (see also subsection 5.2 below).

3.3 The charged sector

One selects in the kinetic terms the terms which give masses to the charged W's when $< H >\neq 0$. For $SU(2)_L$ alone, the quadruplet $(\mathbb{S}^0, \vec{\mathbb{P}})(\mathbb{D}_1, \theta_c)$, which includes the Higgs boson, is the only one concerned since

$$\begin{aligned}
\mathbb{T}^+_L.\mathbb{P}^-(\mathbb{D}_1, \theta_c) &\ni -\frac{i}{\sqrt{2}}\mathbb{S}^0(\mathbb{D}_1) + \ldots \\
\mathbb{T}^-_L.\mathbb{P}^+(\mathbb{D}_1, \theta_c) &\ni -\frac{i}{\sqrt{2}}\mathbb{S}^0(\mathbb{D}_1) + \ldots .
\end{aligned} \tag{26}$$

However, this quadruplet is not stable by the action of $SU(2)_R$ for $\varphi \neq \theta_c$, and gets mixed with $(\mathbb{S}^0, \vec{\mathbb{P}})(\mathbb{D}_4, \theta_c)$. More explicitly, one gets (keeping the terms depending on $\mathbb{S}^0(\mathbb{D}_4)$ for the eventuality when it gets a non-vanishing VEV too –see subsection 3.5 below–)

$$\begin{aligned}
\mathbb{T}^+_R.\mathbb{P}^-(\mathbb{D}_1, \theta_c) &\ni \frac{i}{\sqrt{2}}\left(\cos(\varphi-\theta_c)\mathbb{S}^0(\mathbb{D}_1) + \sin(\varphi-\theta_c)\mathbb{S}^0(\mathbb{D}_4)\right) + \ldots \\
\mathbb{T}^-_R.\mathbb{P}^+(\mathbb{D}_1, \theta_c) &\ni \frac{i}{\sqrt{2}}\left(\cos(\varphi-\theta_c)\mathbb{S}^0(\mathbb{D}_1) - \sin(\varphi-\theta_c)\mathbb{S}^0(\mathbb{D}_4)\right) + \ldots \\
\mathbb{T}^+_R.\mathbb{P}^-(\mathbb{D}_4, \theta_c) &\ni \frac{i}{\sqrt{2}}\left(-\sin(\varphi-\theta_c)\mathbb{S}^0(\mathbb{D}_1) + \cos(\varphi-\theta_c)\mathbb{S}^0(\mathbb{D}_4)\right) + \ldots \\
\mathbb{T}^-_R.\mathbb{P}^+(\mathbb{D}_4, \theta_c) &\ni \frac{i}{\sqrt{2}}\left(\sin(\varphi-\theta_c)\mathbb{S}^0(\mathbb{D}_1) + \cos(\varphi-\theta_c)\mathbb{S}^0(\mathbb{D}_4)\right) + \ldots
\end{aligned} \tag{27}$$

One finds in $D_\mu\mathbb{P}^+(\mathbb{D}_1, \theta_c) \otimes D^\mu\mathbb{P}^-(\mathbb{D}_1, \theta_c) - D_\mu\mathbb{P}^+(\mathbb{D}_4, \theta_c) \otimes D^\mu\mathbb{P}^-(\mathbb{D}_4, \theta_c)$ the mass terms for the charged gauge bosons

$$\frac{v^2}{8}\begin{pmatrix} W^-_{\mu L} & W^-_{\mu R}\end{pmatrix}\begin{pmatrix} g_L^2 & -g_L g_R\cos(\varphi-\theta_c) \\ -g_L g_R\cos(\varphi-\theta_c) & g_R^2\end{pmatrix}\begin{pmatrix} W^+_{\mu L} \\ W^+_{\mu R}\end{pmatrix} \tag{28}$$

which corresponds to the two eigenvalues

$$M_W^2 = \frac{1}{2} M_Z^2 \left(1 \pm \sqrt{1 - 4\sin^2\theta_W \cos^2\theta_W \sin^2(\varphi - \theta_c)}\right). \tag{29}$$

Owing to the constraint that one of the eigenvalues has to match the masses of the observed $W^\pm_{\mu L}$'s

$$M^2_{W^\pm_{\mu L}} = g_L^2 \frac{v^2}{16} = \cos^2\theta_W M_Z^2 \tag{30}$$

one gets

$$\varphi = \theta_c + \frac{\pi}{2}, \tag{31}$$

in which case the $W_L - W_R$ mixing vanishes and the mass eigenstates are the electroweak eigenstates $W^\pm_{\mu L}$ and $W^\pm_{\mu R}$, with

$$M^2_{W^\pm_{\mu R}} = g_R^2 \frac{v^2}{16} = \sin^2\theta_W M_Z^2 \approx (43\, GeV)^2. \tag{32}$$

The case when one allows $< \mathbb{S}^0(\mathbb{D}_4) > \neq 0$ in addition to $< \mathbb{S}^0(\mathbb{D}_1) > \neq 0$ is a straightforward generalization which leads to the same spectrum for the gauge bosons: the only modification is that, now, $v^2/2 = < \mathbb{S}^0(\mathbb{D}_1) >^2 - < \mathbb{S}^0(\mathbb{D}_4) >^2$.

An immediate consequence of (25) and (32) is that, while at high energies (in the W_μ propagator, M_W^2 can be neglected with respect to q^2), the right-handed electroweak interactions are weaker that the left-handed ones because $g_R < g_L$, their effective strengths are identical in the infrared regime $q^2 \ll M_W^2$

$$\left(\frac{g_L}{8M_{W_L}}\right)^2 = \left(\frac{g_R}{8M_{W_R}}\right)^2 = \frac{e^2}{8\sin^2\theta_W \cos^2\theta_W M_Z^2} = \frac{G_F}{\sqrt{2}}, \tag{33}$$

showing, in the charged sector, left-right symmetry as a low energy phenomenon.

3.4 Composite representations of $\mathbf{SU(2)_R}$

The condition (31) enables the $J = 0$ composite representations (14) of $SU(2)_R$ to be rewritten as the quadruplets

$$\begin{aligned}
\phi_{1R}, \tilde\phi_{1R} &= \left[\mathbb{M}^0(\mathbb{D}_1), \mathbb{M}^3(\mathbb{D}_1), \mathbb{M}^+(\mathbb{D}_4, \theta_c), \mathbb{M}^-(\mathbb{D}_4, \theta_c)\right],\\
\phi_{2R}, \tilde\phi_{2R} &= \left[\mathbb{M}^0(\mathbb{D}_4), \mathbb{M}^3(\mathbb{D}_4), \mathbb{M}^+(\mathbb{D}_1, \theta_c), \mathbb{M}^-(\mathbb{D}_1, \theta_c)\right],\\
\phi_{3R}, \tilde\phi_{3R} &= \left[\mathbb{M}^0(\mathbb{D}_2, \theta_c), \mathbb{M}^3(\mathbb{D}_2, \theta_c), \mathbb{M}^+(\mathbb{D}_3, \theta_c), \mathbb{M}^-(\mathbb{D}_3, \theta_c)\right],\\
\phi_{4R}, \tilde\phi_{4R} &= \left[\mathbb{M}^0(\mathbb{D}_3, \theta_c), \mathbb{M}^3(\mathbb{D}_3, \theta_c), \mathbb{M}^+(\mathbb{D}_2, \theta_c), \mathbb{M}^-(\mathbb{D}_2, \theta_c)\right],
\end{aligned} \tag{34}$$

where, as usual, to a scalar $\mathbb{M}^0$ are associated three pseudoscalar $\vec{\mathbb{M}}$'s, and vive-versa. We have emphasized above that, $\mathbb{M}^{0,3}(\mathbb{D}_1, \mathbb{D}_4)$ do not depend on the mixing angle.

3.5 Breaking $\mathbf{SU(2)_L \times SU(2)_R}$

We have only supposed up to now that two scalar fields, $\mathbb{S}^0(\mathbb{D}_1)$ and $\mathbb{S}^0(\mathbb{D}_4)$ eventually get non-vanishing vacuum expectation values, achieving the spontaneous breaking of $SU(2)_L \times SU(2)_R$ down to electromagnetic $U(1)_{em}$.

From (34) the simplest natural potential invariant by $SU(2)_R \times SU(2)_L$ which triggers $< \mathbb{S}^0(\mathbb{D}_1) >\neq 0$ and / or $< \mathbb{S}^0(\mathbb{D}_4) >\neq 0$, breaking it down to $U(1)_{em}$, is

$$V_1 = -\frac{\sigma^2}{2}\left((\mathbb{S}^0, \vec{\mathbb{P}})^{\otimes 2}(\mathbb{D}_1, \theta_c) - (\mathbb{S}^0, \vec{\mathbb{P}})^{\otimes 2}(\mathbb{D}_4, \theta_c)\right) + \frac{\lambda}{4}\left((\mathbb{S}^0, \vec{\mathbb{P}})^{\otimes 2}(\mathbb{D}_1, \theta_c) - (\mathbb{S}^0, \vec{\mathbb{P}})^{\otimes 2}(\mathbb{D}_4, \theta_c)\right)^{\otimes 2}, \tag{35}$$

which has a non-trivial minimum for

$$< \mathbb{S}^0(\mathbb{D}_1)^{\otimes 2} - \mathbb{S}^0(\mathbb{D}_4)^{\otimes 2} >=< \mathbb{S}^0(\mathbb{D}_1)^\dagger \otimes \mathbb{S}^0(\mathbb{D}_1) + \mathbb{S}^0(\mathbb{D}_4)^\dagger \otimes \mathbb{S}^0(\mathbb{D}_4) >\neq 0. \tag{36}$$

Since $< \mathbb{S}^0(\mathbb{D}_4) >\neq 0$ allows for an eventual spontaneous violation of CP, it is natural to consider $| < \mathbb{S}^0(\mathbb{D}_4) > | \ll | < \mathbb{S}^0(\mathbb{D}_1) > |$. One also imposes that no pseudoscalar can condensate in the vacuum $< \vec{\mathbb{P}}(\mathbb{D}_{1\cdots4} >= 0$ (this condition can eventually be relaxed since pseudoscalar condensates can a priori be generated by parity violating electroweak corrections).

Then, the six $\vec{\mathbb{P}}(\mathbb{D}_1), \vec{\mathbb{P}}(\mathbb{D}_4)$ are classically massless. For $< \mathbb{S}^0(\mathbb{D}_4) >\approx 0$ the Higgs boson becomes $h_1 \approx \mathbb{S}^0(\mathbb{D}_1) - < \mathbb{S}^0(\mathbb{D}_1) >$ with mass $M_H^2 = \lambda v^2$, while $h_4 = \mathbb{S}^0(\mathbb{D}_4) - < \mathbb{S}^0(\mathbb{D}_4) >$ is classically massless. Since there are only five massive gauge bosons $Z_\mu, W_{\mu L}^\pm, W_{\mu R}^\pm$, $\mathbb{P}^3(\mathbb{D}_4)$ and h_4 stay classically massless. They however couple to a pair $W_{\mu L} W_{\mu R}$ with a coupling proportional to $g_L g_R < \mathbb{S}^0(\mathbb{D}_1) >$ and can acquire a small mass at the quantum level [6].

More general symmetry breaking potentials involving other quadruplets can be used, which in general increases the number of pseudo-goldstone bosons (see next section).

4 Second case: the "inverted" $\mathbf{SU(2)_R}$

The first chiral extension of the Standard Model studied above is very rigid: because the mixing angle of the right sector is fixed by (31), all couplings of gauge fields to $J = 0$ mesons are fixed too , with reduced hope to fit to experimental constraints. And, indeed, as will be shown in the last section, the first extension above does not seem to be a suitable one.

This is why we now investigate the second possibility $SU(2)_R = \mathcal{G}_{2R}$ with generators given by (8) in which, in particular, no constraint arises for the mixing angle in the right sector, allowing the tuning of the $W_{\mu R}^\pm$ gauge bosons couplings.

The equivalent of the Cabibbo matrix is $\mathbb{D}_2\mathbb{C}$ (with determinant -1) and the $J = 0$ (sum of (2) and $(\overline{2})$ doublets) composite representations of $\mathcal{G}_{2R}$ are the quadruplets

$$\begin{aligned}
\chi_{1R}, \tilde{\chi}_{1R} &= \left[\mathbb{M}^0(\mathbb{D}_1), \mathbb{M}^3(\mathbb{D}_1), \mathbb{M}^+(\mathbb{D}_2, \varphi), \mathbb{M}^-(\mathbb{D}_2, \varphi)\right],\\
\chi_{2R}, \tilde{\chi}_{2R} &= \left[\mathbb{M}^0(\mathbb{D}_2, \varphi), \mathbb{M}^3(\mathbb{D}_2, \varphi), \mathbb{M}^+(\mathbb{D}_1, \varphi), \mathbb{M}^-(\mathbb{D}_1, \varphi)\right],\\
\chi_{3R}, \tilde{\chi}_{3R} &= \left[\mathbb{M}^0(\mathbb{D}_3, \varphi), \mathbb{M}^3(\mathbb{D}_3, \varphi), \mathbb{M}^+(\mathbb{D}_4, \varphi), \mathbb{M}^-(\mathbb{D}_4, \varphi)\right],\\
\chi_{4R}, \tilde{\chi}_{4R} &= \left[\mathbb{M}^0(\mathbb{D}_4), \mathbb{M}^3(\mathbb{D}_4), \mathbb{M}^+(\mathbb{D}_3, \varphi), \mathbb{M}^-(\mathbb{D}_3, \varphi)\right],
\end{aligned} \tag{37}$$

all of them being either of the type $\chi = (\mathbb{S}^0, \vec{\mathbb{P}})$ or $\tilde{\chi} = (\mathbb{P}^0, \vec{\mathbb{S}})$. We have again emphasized in (37) that $\mathbb{M}^{0,3}(\mathbb{D}_1, \mathbb{D}_4)$ in fact do not depend on φ.

One takes for the kinetic terms the same expression deduced from the invariant $\mathcal{J}$ (17) as in the first extension, since $\mathcal{J}$, which is diagonal in the basis of flavour eigenstates (all dependence on the mixing angles disappears then), can equally be expressed in terms of the χ's and $\tilde{\chi}$'s.

4.1 The neutral gauge bosons

Nothing is changed with respect to the discussion made in the previous case, because $\mathbb{T}_2^3 \equiv \mathbb{T}_1^3$, and one gets again eqs. (22), (23), (24) and (25). The $Z_\mu - W_{\nu R}^+ - W_{\rho R}^-$ coupling is also the same as previously.

[6] It is not our subject here but the structure of composite representations of $SU(2)_L$ and $SU(2)_R$ allows to write simple invariant mass terms for the other mesons not involved in the symmetry breaking potential [1].

4.2 The charged sector

Choosing again the Higgs field to be $H = \mathbb{S}^0(\mathbb{D}_1)$, it is simple matter to realize from (37) that the mass terms for the charged gauge fields are again diagonal (no $W_L - W_R$ mixing); they are generated by acting on charged mesons with left or right generators such that the result of this action is $\mathbb{S}^0(\mathbb{D}_1)$; the $W^\pm_{\mu L}$'s get their masses when acting with $\mathbb{T}^\pm_L$ on $\mathbb{P}^\mp(\mathbb{D}_1)$ and the $W^\pm_{\mu R}$'s when acting with $\mathbb{T}^\pm_R$ on $\mathbb{P}^\mp(\mathbb{D}_2)$. Thus, if one writes the Lagrangian in the basis of the $\chi, \tilde\chi$'s, $(1/2)D_\mu\chi_1 D^\mu\chi_1^\dagger$ gives masses to $W^\pm_{\mu R}$ and $(1/2)D_\mu\chi_2 D^\mu\chi_2^\dagger$ to $W^\pm_{\mu L}$, whatever be the value of the mixing angle φ.
One finds, like in the replica case, that

$$M_{W^\pm_{\mu L}} = \cos\theta_W M_Z, \quad M_{W^\pm_{\mu R}} = \sin\theta_W M_Z; \tag{38}$$

the relation (33) implementing left-right symmetry at low energy is still verified.

The difference is that, now, the mixing angle φ is left arbitrary and, in particular, has no connection with the Cabibbo angle.

4.3 Breaking $\mathbf{SU(2)_L \times SU(2)_R}$ down to $\mathbf{U(1)_{em}}$

From (37), a natural $SU(2)_R$ invariant potential to trigger $< \mathbb{S}^0(\mathbb{D}_1) >\neq 0$ is

$$\begin{aligned} V_{2R} &= \chi_{1R}^{\otimes 2} + \chi_{2R}^{\otimes 2} \\ &= -\frac{\sigma^2}{2}\left((\mathbb{S}^0,\vec{\mathbb{P}})^{\otimes 2}(\mathbb{D}_1,\varphi) + (\mathbb{S}^0,\vec{\mathbb{P}})^{\otimes 2}(\mathbb{D}_2,\varphi)\right) + \frac{\lambda}{4}\left((\mathbb{S}^0,\vec{\mathbb{P}})^{\otimes 2}(\mathbb{D}_1,\varphi) + (\mathbb{S}^0,\vec{\mathbb{P}})^{\otimes 2}(\mathbb{D}_2,\varphi)\right)^{\otimes 2} \end{aligned} \tag{39}$$

but, for $\varphi \neq \theta_c$, it is not invariant by $SU(2)_L$; in this general case, it appears that $\mathcal{J}$ (17) is the only expression quadratic in the fields which is left-right invariant; one is accordingly led to introduce the scalar potential

$$\mathcal{V} = -\frac{\sigma^2}{2}\mathcal{J} + \frac{\lambda}{4}\mathcal{J}^{\otimes 2}. \tag{40}$$

$\mathcal{J}$, and thus $\mathcal{V}$ too, is invariant by the full chiral group $U(N)_L \times U(N)_R$ [7] .
If one imposes

$$< \mathbb{P}^{0,1,2,3}(\mathbb{D}_{1\cdots 4}) >= 0 \tag{41}$$

$\mathcal{V}$ has a non-trivial minimum for

$$\begin{aligned} &< \mathbb{S}^0(\mathbb{D}_1)^{\otimes 2} + \mathbb{S}^0(\mathbb{D}_2)^{\otimes 2} + \mathbb{S}^0(\mathbb{D}_3)^{\otimes 2} - \mathbb{S}^0(\mathbb{D}_4)^{\otimes 2} \\ &\qquad -\mathbb{S}^3(\mathbb{D}_1)^{\otimes 2} - \mathbb{S}^3(\mathbb{D}_2)^{\otimes 2} - \mathbb{S}^3(\mathbb{D}_3)^{\otimes 2} + \mathbb{S}^3(\mathbb{D}_4)^{\otimes 2} >= \frac{v^2}{2} \neq 0. \end{aligned} \tag{42}$$

The above combination of neutral scalars does not depend on the mixing angle and can be evaluated with quadruplets of $SU(2)_L$ or $SU(2)_R$.

We are led not to limit ourselves to the case when only $< \mathbb{S}^0(\mathbb{D}_1) >\neq 0$ but to consider the general case

$$\begin{aligned} &< \mathbb{S}^0(\mathbb{D}_1) >\neq 0, < \mathbb{S}^0(\mathbb{D}_2) >\neq 0, < \mathbb{S}^0(\mathbb{D}_3) >\neq 0, < \mathbb{S}^0(\mathbb{D}_4) >\neq 0, \\ &< \mathbb{S}^3(\mathbb{D}_1) >\neq 0, < \mathbb{S}^3(\mathbb{D}_2) >\neq 0, < \mathbb{S}^3(\mathbb{D}_3) >\neq 0, < \mathbb{S}^3(\mathbb{D}_4) >\neq 0, \end{aligned} \tag{43}$$

[7] To demonstrate this, it is convenient to work in the basis of $\mathcal{P}_{even}$ and $\mathcal{P}_{odd}$ [1] flavour eigenstates, and operate on them with the $U(N)$ generators expressed in this same basis (having only one non-vanishing entry equal to 1).

with the $\mathbb{S}^0$'s and $\mathbb{S}^3$'s belonging indifferently to left (depending on θ_c) or right (depending on φ) quadruplets. (43) eventually switches on spontaneous CP violation [8] . Accidental additional invariance of the vacuum in the broken phase may arise.

At the minimum of the potential, the mass terms for all pseudoscalars and charged scalars vanish (we suppose that $<\mathbb{M}_i\mathbb{M}_j> = <\mathbb{M}_i><\mathbb{M}_j>,\ \forall \mathbb{M}_{i,j}$). The spectrum of neutral scalars is obtained by diagonalizing their mass matrix $\mathcal{M}^2$, the entries of which are the corresponding products of vacuum expectations values, for example

$$\mathcal{M}^2_{\mathbb{S}^0(\mathbb{D}_1)\mathbb{S}^0(\mathbb{D}_2)} = \lambda <\mathbb{S}^0(\mathbb{D}_1)><\mathbb{S}^0(\mathbb{D}_2)> . \tag{44}$$

The eigenvalues of $\mathcal{M}^2$, independent of the mixing angles θ_c and φ, are all vanishing but one equal to $M_h^2 = \lambda v^2$. In the special case when only $<\mathbb{S}^0(\mathbb{D}_1)> \neq 0$, the Higgs boson is $h = h_1 \equiv \mathbb{S}^0(\mathbb{D}_1) - <\mathbb{S}^0(\mathbb{D}_1)>$.

The mesonic spectrum which then arises is the following:
- 1 neutral and 4 charged pseudoscalar goldstones which get absorbed by the 5 massive gauge bosons;
- $N^2 - 5 = 11$ pseudoscalars pseudo-goldstone bosons;
- one Higgs boson h with mass $M_h^2 = \lambda v^2$;
- $N^2 - 1 = 15$ scalar pseudo-goldstone bosons.

(43) corresponds to the spontaneous breaking of $U(N)_L \times U(N)_R$ down to $U(1)_{em}$. Because the kinetic and gauge terms in the Lagrangian do not have the full $U(N)_L \times U(N)_R$ but only the $SU(2)_L \times SU(2)_R$ chiral invariance, among the $2N^2 - 1$ goldstone bosons expected in the breaking triggered by $\mathcal{V}$, only five true goldstones, which correspond to the breaking of the gauge subgroup $SU(2)_L \times SU(2)_R$, are eaten by the five massive gauge fields; the other $2N^2 - 6$ are pseudo-goldstones which are expected to get massive only by quantum effects, which makes likely a hierarchy between the electroweak / Higgs mass scales and theirs.

Remark: (43) has consequences on leptonic decays of $J = 0$ mesons mediated by $W^\pm_{\mu L}$ (we neglect the ones mediated by $W^\pm_{\mu R}$, supposing the right-handed neutrinos to be heavy in association with a see-saw mechanism involving a high mass scale); however, up to small deviations which can be accounted for by variations in the leptonic decay constants, the dependence of the decay amplitudes on the Cabibbo angle is very well explained [1] by supposing that $\mathbb{S}^0(\mathbb{D}_1)$ is the only scalar condensing in the vacuum and that, accordingly, $h = h_1$. One is thus led to consider that all other scalar condensates are much smaller than $<\mathbb{S}^0(\mathbb{D}_1)>$.

5 Detection of $\mathbf{W}^\pm_{\mu\mathbf{R}}$

We show here that the charged gauge fields of the right sector are likely to have escaped detection.

5.1 Hadronic colliders

If $W^\pm_{\mu R}$ exist with a mass $\approx 43\,GeV$, they have to be produced in hadronic interactions and specially at proton colliders. However past experiments cannot have detected them; indeed, the decay of $W^\pm_{\mu R}$ into two jets was only investigated by the UA2 experiment [8] at CERN; unfortunately, their lower threshold was $M_{2\,jets} > 48\,GeV$.

Let us also calculate the approximate width of $W^\pm_{\mu R}$.

The picture that we have adopted here has given up the quarks as fundamental fields; however, since the starting $SU(2)_L \times U(1)$ Lagrangian for mesons is built with full compatibility with the standard model for quarks, it is legitimate to consider that its chiral extension should also have its counterpart

[8]It is trivial to restore all signs to be positive in (42), (43) by introducing (or removing) suitable "i" factors.

at the quark level, at least as far as the couplings of the charged gauge bosons are concerned [9]. We accordingly consider that the two $SU(2)_R$ groups $\mathcal{G}_{1R}$ and $\mathcal{G}_{2R}$ provide the two natural chiral extensions of the Glashow-Salam-Weinberg model for quarks in the charged current sector, with Ψ (11) in their fundamental representation.

A good estimate of the width of $W^{\pm}_{\mu L}$ is obtained from the one-loop self-energy from leptons and quarks, and, for the latter, considering only the u and d quarks provides a reasonable approximation (which is all the more valid in the right sector as the top quark is then above threshold):

$$\Gamma^{total}_{W^{\pm}_L} \approx (\sqrt{2}/\pi) G_F M^3_{W_L} \equiv \frac{g_L^2}{4\pi} M_{W_L} \approx 2.5\, GeV. \tag{45}$$

One gets an upper bound for the width of the $W^{\pm}_{\mu R}$ gauge bosons by scaling the previous formula using the relations $M_{W_R} = (\sin\theta_W/\cos\theta_W) M_{W_L}$ and $g_R = (\sin\theta_W/\cos\theta_W) g_L$:

$$\Gamma^{total}_{W_R} \leq \frac{g_R^2}{4\pi} M_{W_R} \equiv \left(\frac{\sin\theta_W}{\cos\theta_W}\right)^3 \Gamma^{total}_{W_L} \approx 200\, MeV; \tag{46}$$

indeed, the leptonic decays should be subtracted from this estimate, which also supposes that the mixing angle to the first generation of quarks is the Cabibbo angle; for $\mathcal{G}_1$, it is clearly an overestimate since $\sin\theta_c$ is now involved instead of $\cos\theta_c$ for the left gauge bosons; for $\mathcal{G}_2$, this amounts to taking $\cos\varphi = \cos\theta_c \approx .975$, very close to its absolute upper bound, which is good enough for our purposes.

(46) is clearly a very small value for $\Gamma^{total}_{W_R}$

$$\frac{\Gamma^{total}_{W_R}}{M_{W_R}} \approx 4.6\, 10^{-3}, \quad \ll \quad \frac{\Gamma^{total}_{W_L}}{M_{W_L}} \approx 2.5\, 10^{-2}, \tag{47}$$

making all the more difficult an eventual detection.

Hadron colliders are thus certainly not a good place to look for the $W^{\pm}_{\mu R}$'s.

5.2 Pair production

As mentioned previously (see subsections 3.2, 4.1), the coupling of a pair of W_R's to the massive Z_μ is damped by a factor $\tan^2\theta_W$ with respect to its equivalent for W_L's; to this damping has to be added the one occurring in leptonic decays of the W_R's because of the presumably heaviness of the right-handed neutrinos, advocating for a see-saw mechanism [9] with another very high mass scale; this concurs to make the detection of a pair of W_R's in e^+e^- colliders very difficult and unlikely.

5.3 Electroweak $\mathbf{SU(2)_R}$ interactions

One must next investigate whether the right gauge bosons can be detected through specific electroweak processes.

5.3.1 Hadronic decays of pseudoscalar mesons

Since the $W^{\pm}_{\mu R}$ are presumably not expected to be detectable through their leptonic decays because of the heavy masses of right-handed neutrinos, one should consider hadronic processes.

Now, the couplings of the right gauge bosons to flavour eigenstates (that we consider, like in [1], to be the asymptotic states) depend on a mixing angle which can be, in the case of replica $SU(2)_R$, $\varphi = \theta_c + \pi/2$ and, in the case of inverted $SU(2)_R$, an arbitrary φ to be determined experimentally like the Cabibbo angle for left-handed interactions.

[9] The case of neutral current is more subtle since $U(1)_{\mathbb{I}}$ gets involved as soon as fermions are concerned.

We shall be concerned with the decays of the type $P \to P_1 P_2 P_3$ where the P's are pseudoscalar mesons. In the quark picture the corresponding diagrams can be cast into two subsets: the "factorizable" ones (fig. 1), which can be divided into two disconnected parts by cutting the internal $W_\mu^\pm$ line, and the "non-factorizable" ones (fig. 2), which cannot.

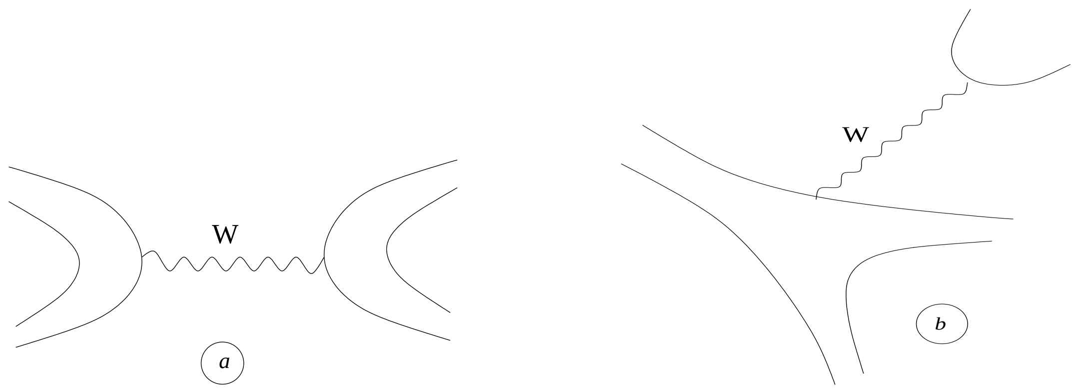

Fig. 1: Factorizable contributions to $P \to P_1 P_2 P_3$ in the quark picture.

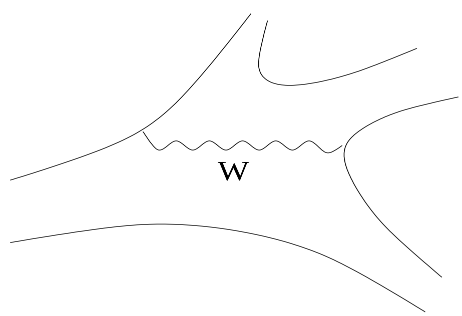

Fig. 2: Non-factorizable contributions to $P \to P_1 P_2 P_3$ in the quark picture.

Most decays involve both factorizable and non-factorizable contributions, for which an eventual cancellation between W_L and W_R is difficult to evaluate; indeed, in the quark picture, one is led to introducing intricate "QCD" corrections, and, in our approach, a new type of "strong-like" interaction like in [10], which, in the case of a 3-body final states like the present one, enormously increases the number of diagrams to take into account.

We are thus inclined to look for decays which involve only factorizable contributions. This is the case for the decay

$$D^+ \to \pi^0 K^+ \pi^0 \tag{48}$$

which is described respectively at the quark level by the diagrams of fig. 1 and in our model by fig. 3.

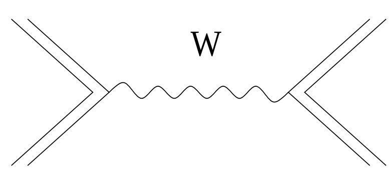

Fig. 3: The decay $P \to P_1 P_2 P_3$ in the present model for $J = 0$ mesons.

There are two types of factorizable diagrams: the first (fig. 1a) would correspond to the formation of a K^* resonance then decaying into $K\pi$, and the second (fig. 1b) to that of a ρ resonance which yields two π's; for the latter, the outgoing K is akin to the longitudinal part of the massive $W_\mu^\pm$ [1].

It seems legitimate to make a one-to-one correspondence between the factorizable diagrams of fig. (1a) and the ones of fig. (3) which naturally arise, in the proposed extension of the standard model to $J = 0$, from the derivative couplings of one gauge fields to two pseudoscalars; the transcription of the diagrams of figs. (1b) and (2) needs introducing another type of interactions [10].

While the $\rho^0 K^+$ channel is mentioned, the $\pi^0 K^+ \pi^0$ does not appear in the Table of Particle Properties [10] .

We interpret this as the absence of the channel associated with fig. 3, and attribute it to a cancellation between the contributions of $W_{\mu L}^\pm$ and $W_{\mu R}^\pm$; this distinguishes between the two types of chiral extensions that we proposed. Staying in the low energy limit where one can neglect the momentum dependence of the $W_\mu^\pm$ propagator, it is trivial matter to deduce, from the expression of the generators

$$\mathbb{T}_L^+(\varphi) = \left(\begin{array}{c|cc} & \cos\theta_c & \sin\theta_c \\ & -\sin\theta_c & \cos\theta_c \\ \hline & & \end{array}\right), \tag{49}$$

$\mathbb{T}_{1R}^+(\varphi = \theta_c + \pi/2)$ given by (7) and $\mathbb{T}_{2R}^+(\varphi)$ given by (8) that the dependence of the amplitude on the mixing angle, which is $\sin^2\theta_c$ for $W_L^\pm$ alone becomes:
- $\sin^2\theta_c + \cos^2\theta_c = 1$ if one takes $SU(2)_R \equiv \mathcal{G}_1$;
- $\sin^2\theta_c - \sin^2\varphi$ if one takes $SU(2)_R \equiv \mathcal{G}_2$.

This decay would thus be strongly enhanced if $SU(2)_R \equiv \mathcal{G}_1$, since there is no more Cabibbo suppression; we are thus led to favour the inverted $SU(2)_R$ as the preferred extension of our model; small values of the mixing angle φ are obviously favoured to avoid a large contribution from the right sector; this statement will get strengthened in the following subsection.

5.3.2 The $\mathbf{K_L - K_S}$ mass difference

The most stringent lower bounds for the mass of $W_{\mu R}^\pm$ come from their contribution to the box diagrams controlling the $K_L - K_S$ mass difference [5]. They have however been obtained with restrictive hypothesis concerning in particular the coupling and mixing angle in the right sector. Different conclusions can be reached in the framework proposed above.

We consider here again that the extension that we proposed for $J = 0$ mesons can be transcribed at the level of quarks in the most intuitive way for charged currents.

Including the contributions of $W_{\mu R}^\pm$, the box diagrams evaluated with u and c quarks [11] , in the 't Hooft-Feynman gauge [12] and neglecting the momenta of external quarks for the gauge fields, yield the amplitude

$$\mathcal{A}(d\bar{s} \to s\bar{d}) = \mathcal{A}_{LL}\left(1 + \frac{\sin^2\varphi\cos^2\varphi}{\sin^2\theta_c\cos^2\theta_c}\frac{\mathcal{O}_{RR}}{\mathcal{O}_{LL}} + \frac{\sin\varphi\cos\varphi}{\sin\theta_c\cos\theta_c}\frac{(M_{W_L}^2/M_{W_R}^2)\ln(M_{W_L}^2/M_{W_R}^2)}{1 - M_{W_L}^2/M_{W_R}^2}\frac{\mathcal{O}_{LR}}{\mathcal{O}_{LL}}\right), \tag{50}$$

[10] There only appears the similar doubly Cabibbo suppressed decay $D^+ \to K^+\pi^+\pi^-$ which corresponds in the quark model to non-factorizable contributions (see fig. 2).

[11] The role of the top quark is expected to be small as far as the $K_L - K_S$ mass difference is concerned, because of the behaviour of the corresponding mixing angles [11]

[12] They have been shown, in the quest for gauge invariance [12], to yield the dominant contribution.

where $\mathcal{A}_{LL}$ corresponds to the standard result with two $W^{\pm}_{\mu L}$'s

$$\mathcal{A}_{LL} = \frac{G_F^2}{4\pi^2} m_c^2 \sin^2\theta_c \cos^2\theta_c \,\mathcal{O}_{LL}, \tag{51}$$

the second term in the parenthesis corresponds to the contributions of two $W^{\pm}_{\mu R}$'s, the third term to the $W^{\pm}_{\mu L} - W^{\pm}_{\mu R}$ crossed contributions, and $\mathcal{O}_{LL}$, $\mathcal{O}_{RR}$ and $\mathcal{O}_{LR}$ are respectively the operators

$$\begin{aligned}
\mathcal{O}_{LL} &= [\bar{s}\gamma_\mu(1-\gamma_5)d]\,[\bar{s}\gamma_\mu(1-\gamma_5)d],\\
\mathcal{O}_{RR} &= [\bar{s}\gamma_\mu(1+\gamma_5)d]\,[\bar{s}\gamma_\mu(1+\gamma_5)d],\\
\mathcal{O}_{LR} &= [\bar{s}(1-\gamma_5)d]\,[\bar{s}(1+\gamma_5)d],
\end{aligned} \tag{52}$$

the matrix elements of which between $\bar{K}^0$ and K^0 we approximate as usual by inserting the vacuum as the intermediate state and using $PCAC$; this leads to

$$< \bar{K}^0|\mathcal{O}_{RR}|K^0 > = < \bar{K}^0|\mathcal{O}_{LL}|K^0 > \tag{53}$$

and to [5]

$$< \bar{K}^0|\mathcal{O}_{LR}|K^0 > \approx 7.7 < \bar{K}^0|\mathcal{O}_{LL}|K^0 > . \tag{54}$$

One finally gets for $M_{W_{\mu L}} \approx 80\, GeV$ and $M_{W_{\mu R}} \approx 43\, GeV$

$$\mathcal{A}(K^0 \rightarrow \bar{K}^0) \approx \mathcal{A}_{LL}(K^0 \rightarrow \bar{K}^0)\left(1 + \frac{\sin^2\varphi\cos^2\varphi}{\sin^2\theta_c\cos^2\theta_c} - 13.45\frac{\sin\varphi\cos\varphi}{\sin\theta_c\cos\theta_c}\right) \tag{55}$$

The corrections due to $W^{\pm}_{\mu R}$ vanish with φ, which is constrained by (55) to

$$\varphi \ll \theta_c, \tag{56}$$

and the mixing angle in the right sector can be chosen small enough for $W^{\pm}_{\mu R}$ to give negligeable contributions to the $K_L - K_S$ mass difference.

5.3.3 $\boldsymbol{\mu \rightarrow e\gamma}$

The smallness of the mixing angles in the right sector can counterbalance as much as desired the small mass of the right gauge bosons and prevent any unwanted enhancement of such decays [4].

5.4 Light but elusive $\mathbf{W^{\pm}_{\mu R}}$'s

We have shown that a chiral completion of the standard electroweak model in the mesonic sector can call for $W^{\pm}_{\mu R}$ gauge bosons much lighter than usually expected which, nevertheless, are likely to have escaped detection.

In the extension favoured by experimental data the two families of right-handed quarks lie in inequivalent representations of $SU(2)_R$ and the associated mixing angle is constrained to be much smaller that the Cabibbo angle.

6 Conclusion

The large arbitrariness in completing the standard model with a right-handed sector was often reduced with arbitrary choices, including the equality of the coupling constants [3] and simple relations between the left and right mixing angles [2]. Also, when dealing with fermions, the $U(1)_{\mathbb{Y}}$ of weak

hypercharge cannot be embedded in a right handed group, making an extra massive Z'_μ gauge field Z'_μ expected, in addition to new massive $W^{\pm}_R$'s. Last, the universality of behaviour for the different families of fermions observed for left-handed interactions were always assumed to be also true for right-handed interactions.

The three points mentioned above have received here somewhat less conventional answers:
- the existence of an extra Z'_μ has been decoupled from the spectrum of the charged $W^{\pm}_{\mu R}$ by working in the (composite) mesonic sector, which includes in particular the Higgs multiplet(s) responsible of the gauge boson masses;
- the right coupling constant is different from the left one, as suggested by the chiral Gell-Mann-Nishijima relation and, in no case, the mixing angles in the right and left sectors appear to match; the former is most likely (still) another arbitrary parameter to be determined from experiment, excluding in particular the cases of a manifest or pseudo-manifest left-right symmetry [2];
- the universality in the behaviour of families for left-handed interactions cannot be taken for granted in the right sector and is likely to be broken in a very specific way if the independence of the left and right mixing angles, yielding the maximum flexibility of the model, is to be achieved (which seems indeed to be the situation favoured by experimental data).

Problems arise when one wants to use this approach for leptons, since the $U(1)_{\mathbb{I}}$ group cannot be concealed any more. A large variety of $SU(2)_L \times SU(2)_R \times U(1)_{\mathbb{I}}$ $(2, \overline{2}, 1)$ multiplets, formed from doublets making up the reduceable composite $SU(2)_L$ and $SU(2)_R$ quadruplets exhibited above, can be used to give masses to the fermions (and then the hierarchy of fermionic masses has to be linked to a hierarchy between different "$q\bar{q}$ condensates"), but a coupling of $W^3_{\mu R}$ to $-(1/2)\bar{\Psi}\gamma^\mu \mathbb{I}\Psi$ which arises if we keep matching B_μ with $W^3_{\mu R}$ in the Gell-Mann-Nishijima relation without further modification explicitly breaks $SU(2)_R$. The perspective of adding an extra Z'_μ [13] appears non trivial: it cannot in particular be coupled to $U(1)_{\mathbb{I}}$ alone without extending the Higgs structure of the model, since none of the quark-antiquark bound states of the type (12) or (14), being invariant by the action of $\mathbb{I}$, can make it massive [14] . We thus have to tackle a whole reconstruction of the extended model, probably also influencing the charged $W^{\pm}_{\mu R}$, suitable for the leptonic sector, but which has to accommodate the outcome of the present study in the (composite) mesonic sector. The challenge of generating reasonable neutrino masses (including having a large scale to trigger a see-saw mechanism as invoked in this work) and avoiding cosmological problems also comes into play.

This will be the subject of a subsequent work. I hope that this limited study has suggested that considering interactions between quark-antiquark composite fields used as reasonable substitutes for $J = 0$ mesons both widens the range of possibilities for extending the standard electroweak model and, together, yields new constraints on them.

Acknowledgements: It is a pleasure to thank M. Banner and M.B. Gavela for stimulating and enlightening discussions.

List of Figures

[13] Many studies have already dealt with the physics of Z'_μ gauge bosons; see for example [13] and references therein.

[14] To give mass to a Z'_μ only coupled to $U(1)_{\mathbb{I}}$ one needs for example to introduce new Higgs multiplets which are not $U(1)_{\mathbb{I}}$ singlets, like $SU(2)$ triplets with nonvanishing $B - L$. These can in particular trigger the wished for see-saw mechanism , but are usually associated with charged $W^{\pm}_{\mu R}$ much heavier than considered here .

References

[1] *B. MACHET: "Chiral scalar fields, electroweak $SU(2)_L \times U(1)$ and the quantization of the electric charge", Phys. Lett. B 385 (1996) 198-20, hep-ph/9606239.*

[2] *See for example:*
P. LANGACKER & S. UMA SANKAR: "Bounds on the mass of W_R and the $W_L - W_R$ mixing angle ζ in general $SU(2)_L \times SU(2)_R \times U(1)$ models", Phys. Rev. D 40 (1989) 1569, and references therein.

[3] *G. SENJANOVIC & R.N. MOHAPATRA: "Exact left-right symmetry and spontaneous violation of parity", Phys. Rev. D 12 (1975) 1502.*

[4] *R.N. MOHAPATRA & G. SENJANOVIC: "Neutrino Mass and Spontaneous Parity Nonconservation", Phys. Rev. Lett. 44 (1980) 912; "Neutrino masses and mixings in gauge models with spontaneous parity violation", Phys. Rev. D 23 (1981) 165.*

[5] *G. BEALL, M. BANDER & A SONI:"Constraint on the Mass Scale of a Left-Right-Symmetric Electroweak Theory from the $K_L - K_S$ Mass Difference", Phys. Rev. Lett. 48 (1982) 848.*

[6] *R.E. MARSHAK & R.N. MOHAPATRA: "Quark-lepton symmetry and $B - L$ as the $U(1)$ generator of the electroweak symmetry group", Phys. Lett. 91B (1980) 222.*

[7] *B. MACHET: "Aspects of electroweak physics for a composite Higgs boson; application to the Z gauge boson decaying into two leptons and two pseudoscalar mesons", hep-ph/9904327.*

[8] *UA2 Collaboration: "A search for new intermediate vector bosons and excited quarks decaying to two-jets at the CERN $p\bar{p}$ collider", Nucl. Phys. B400 (1993) 3.*

[9] *See for example:*
S.M. BILENKY, C. GIUNTI & W. GRIMUS: "Phenomenology of neutrino oscillations", Prog. Part. Nucl. Phys. 43 (1999) 1-86, hep-ph/9812360, and references therein.

[10] *B. MACHET: "Large isospin breaking in an electroweak $SU(2)_L \times U(1)$ gauge theory of $J = 0$ mesons: the case of $K \rightarrow \pi\pi$ decays", Int. J. Mod. Phys. A 14 (1999) 4003-4042, hep-ph/9805458.*

[11] *M.B. GAVELA, B. MACHET & S.T. PETCOV: in preparation.*

[12] *W.-S. HOU & A. SONI: "Gauge invariance of the $K_L - K_S$ mass difference in left-right symmetric models", Phys. Rev. D32 (1985) 163;*
J. BASECQ & L.-F. LI, P.B. PAL: "Gauge-invariant calculation of the $K_L - K_S$ mass difference in the left-right model", Phys. Rev. D32 (1985) 175.

[13] *F. DEL AGUILA: "The Physics of Z' bosons", Lectures given at 27th International School of Theoretical Physics: Standard Model and Beyond '93, Szczyrk, Poland, 19-27 Sep 1993. Published in Acta Phys. Polon. B25 (1994) 1317-1336, hep-ph/9404323;*
K.S. BABU, Ch. KOLDA & J. MARCH-RUSSEL:"Implications of Generalized $Z - Z'$ mixing", Phys. Rev. D57 (1998) 6788-6792, hep-ph/9710441;
A. LEIKE: "The phenomenology of extra neutral gauge bosons", Phys. Rept. 317 (1999) 143-250, hep-ph/9805494;
J. CHAY, K.Y. LEE & S.H. NAM: "Bounds on the mass of Z' and the neutral mixing angles in general $SU(2)_L \times SU(2)_R \times U(1)$ models", Phys. Rev. D61 (2000) 035002, hep-ph/9809298.